A combination of hadronic form factors for modeling
the kaon photoproduction process $\gamma p \rightarrow K^+\Lambda$

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We have phenomenologically investigated the kaon photoproduction process $\gamma p \rightarrow K^+\Lambda$
by combining different types of hadronic form factors (HFFs) inside a covariant isobar model. We obtained the best model with the smallest $\chi^2/N$ by using the dipole form factor in the Born terms and a combination of the dipole, Gaussian, as well as generalized dipole form factors in the hadronic vertices of the nucleon, kaon and hyperon resonances. By utilizing this model we found that the experimental data used in the analysis are internally consistent, whereas the behavior of differential cross-section at forward angles is not significantly affected by the variation of hadronic coupling constants (CCs) and form factor cutoffs in the model.

Keywords: Kaon; hadronic form factor; photoproduction.

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1. Introduction

The existence of hadronic form factor (HFF) in the hadronic vertex of an elementary process is inevitable, since hadron is not a point-like particle. In general, this form factor might be considered as a representation of matter distribution inside the hadron and, as a consequence, a careful and detailed investigation of hadronic processes should always take the HFF into account.

In a previous study, we have phenomenologically investigated the kaon photoproduction process $\gamma p \rightarrow K^+\Lambda$ by using global HFF in all hadronic vertices\(^1\). A global form factor was assumed for the sake of simplicity, because with just a few number of different form factor types the number of possible combinations of utilizing them in the model could be enormous. For instance, in the previous investigation we had 14 different hadronic vertices in the model. With five different types of form factors,
the total possible combinations become $5^{14} = 6,103,515,625$. Performing fits for such a large number of combinations is obviously a daunting task. Furthermore, in view of the accuracy of current experimental data as well as the effort devoted to this task, performing this massive number of fits could be meaningless. To circumvent this problem, we need a certain fitting strategy. It is the purpose of this paper to explain this strategy and explore the result. Moreover, after obtaining the best form factor combination, we can further investigate some related but important aspects, e.g., the internal consistency of the data and the calculated differential cross-section at forward angles. The latter becomes very important nowadays, because it is very decisive for the prediction of the hypernuclear production cross-section.

There has been an ample number of investigations of kaon photoproduction in the literature. Most of the recent investigations\textsuperscript{2–17} have been devoted to search for missing resonances, i.e., the resonances predicted by some quark models but not yet listed by the Particle Data Group (PDG).\textsuperscript{18} This is understandable, since a solid knowledge of the nucleon spectra could provide important information on the structure of the nucleon. On the other hand, this process has been also used to study other phenomenological topics, e.g., the Regge phenomenon,\textsuperscript{19} threshold behavior of the process,\textsuperscript{20} application of the chiral perturbation theory,\textsuperscript{21,22} as well as production of kaon on a deuteron\textsuperscript{23–26} or heavier nuclei.\textsuperscript{27} Nevertheless, the existence of HFFs in this process still receives less attention (see, e.g., Ref.\textsuperscript{28} for a short review), in spite of the fact that the involved particles are hadrons.

Besides the strong support from theoretical perspective, there is an obvious advantage of using HFFs in phenomenological models. It is well-known that the Born amplitude contains the energy-dependent terms which will violently increase as the energy increases. Consequently, the cross-section calculated from such a “bare” amplitude will overshoot the experimental data. Thus, the inclusion of HFF in this case provides an effective suppression of the amplitude in the higher energy regime. Note, however, that a direct inclusion of the form factor will destroy the gauge invariance of the Born terms, since in this case, not all terms are individually gauge invariant. In the literature, we can find a number of recipes that has been put forward to fix this problem.\textsuperscript{29,30} Finally, we also note that there has been criticism of the use of HFF in meson photoproduction, since its role to suppress the divergent amplitude could be replaced by including two selected hyperon resonances, i.e., the $\Lambda(1800)S_{01}$ and $\Lambda(1810)P_{01}$.\textsuperscript{31} Our recent study with more experimental data included, however, indicates that a better result would be obtained by using the HFFs and including the $\Lambda(1600)P_{01}$ and $\Lambda(1810)P_{01}$ hyperon resonances.\textsuperscript{32}

The preliminary result of the findings described in the present paper has been reported in a conference proceedings.\textsuperscript{33} The organization of this paper is as follows: In Sec. 2, we briefly present the elementary model used in this study. Section 3 summarizes the types of HFFs used in the present study. In Sec. 4, we discuss the methods for restoring gauge invariance after the inclusion of HFFs. The results of our investigation will be given in Sec. 5. We will summarize and conclude our findings in Sec. 6.
2. The Elementary Model

In the present study, we use the same covariant isobar model as in our previous studies.\textsuperscript{1,34} The model is constructed from the most suitable Feynman diagrams that describe the background and resonance terms. The background terms consist of the standard $s$, $t$- and $u$-Born channels along with two vector mesons with different parities, i.e., the $K^*(892)$ and $K_1(1270)$, that have been shown to effectively suppress the diverging Born terms. Furthermore, two hyperon resonances, i.e., $\Lambda(1600)P_{01}$ and $\Lambda(1810)P_{01}$, are included in order to increase the HFF cutoffs discussed in detail in Ref. 32.

The resonance part consists of the nucleon resonances taken from PDG,\textsuperscript{18} which have masses between the $K^+\Lambda$ threshold energy (1.609 GeV) and the energy upper limit of the data, i.e., 2.2 GeV. The model does not include the nucleon resonances with masses below the threshold energy, because their couplings were found to be small.\textsuperscript{35} To avoid more uncertainties in the model, as well as to simplify the formalism the resonance spin is limited only up to 3/2. In addition, the model also uses the $P_{13}(1840)$ state, which was found in Ref. 36 to significantly, contribute to the photoproduction of $K^+\Lambda$, $K^+\Sigma^0$ and $K^0\Sigma^+$. In our recent study\textsuperscript{34} we have also obtained the same findings. Thus, in the nucleon resonance terms we use the $N^*(1650)S_{11}$, $N^*(1700)D_{13}$, $N^*(1710)P_{11}$, $N^*(1720)P_{13}$, $N^*(1840)P_{11}$, $N^*(1900)P_{13}$, $N^*(2080)D_{13}$, $N^*(2090)S_{11}$ and $N^*(2100)P_{11}$ states. Since hadron is not point-like, as discussed in Sec. 1, we include HFFs in the hadronic vertices and make use of the method explained in Ref. 30 to restore the gauge invariance. A brief discussion of this topic is given in Sec. 4. For a detailed explanation of this model we refer the reader to Ref. 34. Note that the model is able to nicely reproduce all available experimental data for the $K^+\Lambda$ photoproduction, which consists of more than 3500 data points, with $\chi^2/N = 2.57$ in the original model\textsuperscript{34} and $\chi^2/N = 2.30$ in the modified one.\textsuperscript{1}

3. Hadronic Form Factors

In the previous study,\textsuperscript{1} we have explained in detail the types of HFFs found in the literature. To visualize the ability of the form factors in suppressing the amplitude, we have also plotted their values as functions of the four-momentum squared and their cutoffs. Here, we will just briefly explain the five types of form factors used in the present study. Further explanation can be found in Ref. 1.

The first HFF used in this analysis is the dipole form, which reads

\[ F(q^2) = \frac{\Lambda^4}{\Lambda^4 + (q^2 - m^2)^2} \]  

To our knowledge, this form factor is the most frequently used form factor in the investigation of meson production. In Eq. (1), $\Lambda$ indicates the form factor cutoff, while $q^2$ denotes the four-momentum squared of the off-shell particle whose mass is $m$. This form factor does not have any singularity even at the pole position, i.e.,
For the sake of clarity, the dipole form factor is sometimes written as
\[ F(q^2) = \left\{ 1 + \frac{(q^2 - m^2)^2}{\Lambda^2} \right\}^{-1}, \] (2)
which shows that the form factor is free of poles for a finite value of \( \Lambda \).

The Gaussian form factor is also frequently used in the investigation of meson production since it is free of singularity too. The most commonly used form in this case reads
\[ F(q^2) = \exp\left(-\frac{(q^2 - m^2)^2}{\Lambda^4}\right). \] (3)

As shown in our previous study,\(^1\) this form factor can suppress the amplitude much faster than the monopole or dipole one.

The last form factor used in the present study is the generalized dipole form factor proposed more than three decades ago.\(^{37}\) We have also investigated the effect of this form factor in our previous study. The form factor can be written as
\[ F(x) = \frac{1}{1 + a_1|x| + \cdots + a_r|x|^r}, \] (4)
where \( x \) is the four-momentum squared of the off-shell particle, written as \( q^2 \) in Eq. (1). As in the previous study, we rewrite this form factor as
\[ F(q^2) = \left\{ 1 + \frac{|q^2 - m^2|}{\Lambda^2} + \cdots + \frac{|q^2 - m^2|^r}{\Lambda^2} \right\}^{-1}. \] (5)

Note that in the present analysis the number of terms \( r \) in Eq. (5) is limited up to three. Therefore, in total we have five different types of form factors. Depending on the terms, the four-momentum squared of the off-shell particle \( q^2 \) will be replaced by the Mandelstam variables \( s, t, \) or \( u. \)

4. Restoring the Gauge Invariance

There has been a number of methods proposed in the literature to restore the gauge invariance after including the HFFs.\(^{29,30,38,39}\) For the sake of clarity, we will briefly discuss the application of these methods in kaon photoproduction. Our discussion in this section is primarily based on Refs. 1 and 30.

Figure 1 exhibits the \( s- \) and \( t- \) channel Feynman diagrams that contribute to the background terms of kaon photoproduction with the kinematics convention \( \gamma(p) + p(k) \rightarrow K^+(q) + \Lambda(p_\Lambda). \) It is well-known that both diagrams are not individually gauge invariant and in the case of point particles (no HFFs) only by adding the two contributions we will get the gauge invariant amplitude, i.e.,
\[ \epsilon_\mu J^\mu = \frac{4ieg_\gamma g}{(s - m_K^2)(t - m_K^2)} \bar{u}_\Lambda \{p \cdot k q \cdot \epsilon - p \cdot \epsilon q \cdot k\} u_p, \] (6)
where \( g = g_{KAN} \) is the leading coupling constant (CC) and we have used the pseudoscalar coupling. Note that the gauge invariance implies that the amplitude will
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Fig. 1. (Color online) (a) The $s$-channel and (b) $t$-channel Feynman diagrams for $K^+\Lambda$ photoproduction on the proton that are not individually gauge invariant. The HFFs are indicated in the $s$- and $t$-channel. The contact term (c) is required to restore the gauge invariance after the inclusion of HFFs.

vanish if we replace the photon polarization vector $\epsilon_\mu$ with the photon momentum $k_\mu$.

If we consider the involved hadrons as composite particles, the HFFs automatically appear in all hadronic vertices as indicated in Figs. 1(a) and 1(b). With the definition of Mandelstam variables $s = (k + p)^2$ and $t = (k - q)^2$, as well as $F_t \equiv F(t)$ and $F_s \equiv F(s)$ for the sake of brevity, the amplitude now reads

$$\epsilon_\mu J^\mu = \frac{4ieg_5}{(s - m_\rho^2)(t - m_K^2)} \bar{u}_\Lambda \{ p \cdot k q \cdot \epsilon F_t - p \cdot \epsilon q \cdot k F_s \} u_p,$$

which is obviously no longer gauge invariant, since $F_t \neq F_s$. For the purpose of our present discussion we can recast Eq. (7) to

$$\epsilon_\mu J^\mu = \frac{4ieg_5}{(s - m_\rho^2)(t - m_K^2)} \bar{u}_\Lambda \{ (p \cdot k q \cdot \epsilon - p \cdot \epsilon q \cdot k) \hat{F} - \{ p \cdot k q \cdot \epsilon (\hat{F} - F_t) - p \cdot \epsilon q \cdot k (\hat{F} - F_s) \} \} u_p,$$

where g.i. stands for gauge invariant and in Eq. (8) we have introduced a new form factor $\hat{F}$ for the g.i. term. Note that the non-g.i. term vanishes in the case of point particles, since by definition in this case $\hat{F} = F_s = F_t = 1$.

To restore the gauge invariance one can introduce an additional contact term shown in Fig. 1(c). To this end, all methods proposed in the literature introduce basically an additional contact current, represented by the contact term in Fig. 1(c), which cancels exactly the non-g.i. term given in Eq. (8), i.e.,

$$\epsilon_\mu J^\mu_c = -\epsilon_\mu J^\mu_{\text{non-g.i.}}.$$

In the method proposed by Ohta,\textsuperscript{39} it is found that $\hat{F} = 1$, whereas in the method proposed by Haberzettl,\textsuperscript{38} it is obtained that $\hat{F}$ could be a function of Mandelstam variables $s$, $u$ and $t$. Since this choice will directly affect the g.i. term and, moreover, considering more flexibilities in fitting the parameters provided by the latter method, in the present analysis we use the method proposed by Haberzettl.\textsuperscript{30,38}
with

\[ \hat{F} = a_1 F(s) + a_2 F(u) + a_3 F(t), \]  

(10)

where \(a_1 + a_2 + a_3 = 1\), in order to ensure the correct limit for zero photon momentum.\(^{30}\) In practice, we take

\[ a_1 = \sin^2 \theta_h \cos^2 \phi_h, \]  

(11)

\[ a_2 = \sin^2 \theta_h \sin^2 \phi_h, \]  

(12)

\[ a_3 = \cos^2 \theta_h, \]  

(13)

where \(\theta_h\) and \(\phi_h\) are considered as free parameters.

5. Results and Discussion

5.1. Variation of the HFF cutoff

As discussed in Sec. 1, the number of possible combinations of HFFs used in each hadronic vertex is enormous. Our strategy to overcome this problem is summarized in the following steps.

5.1.1. Step 1. Global type of form factor

In general, we divide the hadronic vertices in the model into four groups, i.e., (i) Born terms, (ii) \(K^*\) terms, (iii) \(Y^*\) terms and (iv) \(N^*\) terms. As a first step, we use the same type of form factor for all terms in the model. Since we have five form factors, there are five possible fit combinations. Note that we refit the same data as in the original model.\(^{34}\) The result is given in Table 1. Obviously, the use of the dipole form factor is recommended by this step.

5.1.2. Step 2. Variation of the group form factor

Having obtained the dipole form factor as the best global form factor, we start with the variation of the form factors in the hadronic vertices of the four groups mentioned above. Since we have five types of form factors and four groups of contributing terms, the total combinations are \(17 \times 5 = 85\). We refit the model for all 85 combinations and found the values of \(\chi^2/N\) in the range between 2.47 and 3.44. For

<table>
<thead>
<tr>
<th>Form factor type</th>
<th>(\chi^2/N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole</td>
<td>2.60</td>
</tr>
<tr>
<td>Gaussian</td>
<td>2.70</td>
</tr>
<tr>
<td>Generalized dipole 1</td>
<td>2.98</td>
</tr>
<tr>
<td>Generalized dipole 2</td>
<td>2.91</td>
</tr>
<tr>
<td>Generalized dipole 3</td>
<td>2.89</td>
</tr>
</tbody>
</table>

Table 1. Values of \(\chi^2/N\) obtained by using global form factor in the \(K^+\Lambda\) photoproduction model.
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Table 2. Values of $\chi^2/N$ obtained by varying the type of HFFs used in the four contributing terms to the $K^+\Lambda$ photoproduction model. In this table the type of form factors is abbreviated as: Dipole (Dip), Gaussian (Gau), Generalized dipole 1 (GD1), Generalized dipole 2 (GD2) and Generalized dipole 3 (GD3). Note that there are 85 possible combinations. Only six combinations are shown here.

<table>
<thead>
<tr>
<th>Born</th>
<th>Nucleon</th>
<th>Meson</th>
<th>Hyperon</th>
<th>$\chi^2/N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GD1</td>
<td>Gau</td>
<td>Gau</td>
<td>Gau</td>
<td>3.44</td>
</tr>
<tr>
<td>GD1</td>
<td>Dip</td>
<td>Dip</td>
<td>Dip</td>
<td>3.43</td>
</tr>
<tr>
<td>Gau</td>
<td>Gau</td>
<td>GD2</td>
<td>Gau</td>
<td>3.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dip</td>
<td>Dip</td>
<td>GD3</td>
<td>Dip</td>
<td>2.49</td>
</tr>
<tr>
<td>Dip</td>
<td>Dip</td>
<td>Gau</td>
<td>Dip</td>
<td>2.48</td>
</tr>
<tr>
<td>Dip</td>
<td>Gau</td>
<td>Gau</td>
<td>Gau</td>
<td>2.47</td>
</tr>
</tbody>
</table>

the sake of simplicity, in Table 2, we only list the results from six form factor combinations which represent the three highest and three lowest $\chi^2/N$ values obtained in the fit process. Other possible combinations clearly yield the $\chi^2/N$ within these values.

From the two last combinations shown in Table 2, we may conclude that the best result would be obtained by using the dipole form factor in the Born terms and the Gaussian one in the $K^*$ terms. Note that the lowest $\chi^2/N$ would be obtained by using Gaussian form factor also in the nucleon and hyperon resonance terms. However, the use of dipole form in the nucleon and hyperon resonances is still acceptable, since the $\chi^2/N$ obtained in this case differs only by 0.01. In most possible combinations, we obtain that the dipole form factor is better used, except for the $K^*$ terms. This result is consistent with the result obtained in the first step.

To further check the consistency of the result given in Table 2, we refit the model by keeping the Gaussian form factor for the $K^*$ terms and vary the form factors in the other three groups. Again, we obtain a consistent result as given in Table 2.

5.1.3. Step 3. Variation of all form factors inside the group

Finally, we vary the type of form factors of the individual resonances inside the groups. For instance, we vary the form factors of the individual nucleon resonances, whereas the form factors for the Born, hyperon and kaon resonances are kept fixed. Fitting with more than 100 form factor combinations have been performed for this purpose. The best result is obtained with $\chi^2/N = 2.13$ and the corresponding form factor for each term is given in Table 3. Note that for consistency we keep here the notation of the original model, which conforms the notation of the previous version of PDG listing,\textsuperscript{46} e.g., the $N(2080)D_{13}$ which has been replaced by the $N(1875)D_{13}$ in the new PDG listing.\textsuperscript{18} Nonetheless, in the best model the mass
the modified model, respectively, close to the values given by the new PDG listing, i.e., 1875 and 165 MeV, respectively. The result is obviously better than the original model ($\chi^2/N = 3.34$) that uses the dipole form as the global HFF and still better than the modified model ($\chi^2/N = 2.30$) that assumes the Gaussian form factor.

Except in the $K^*$ hadronic vertex, Table 3 shows that the combination of dipole and Gaussian form factors in the hadronic vertices of the model yields an effective mechanism to reduce the $\chi^2$ value. This result can be understood because the use of dipole form factor is more preferred by most of the terms in the model as discussed above, whereas the use of Gaussian form factor as the global HFF yields the best model, as shown in our previous study.1 Therefore, an appropriate combination of the dipole and Gaussian form factors would certainly give the best result. The $K^*(892)$ intermediate state seems to require a stronger suppression, i.e., the Gaussian form factor, because its contribution to the amplitude is relatively small.

Table 3. The best HFF combination obtained in this study along with the corresponding form factor cutoffs (\(\Lambda\)) and CCs. Notation for the type of form factors given in the second column is the same as in Table 2. Note that with this HFF combinations we obtain $\chi^2/N = 2.13$.

<table>
<thead>
<tr>
<th>Terms</th>
<th>HFF</th>
<th>(\Lambda) (GeV)</th>
<th>CC</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Born Terms</td>
<td>Dip</td>
<td>1.27</td>
<td></td>
<td>$g_{KNN}/\sqrt{4\pi}$</td>
</tr>
<tr>
<td></td>
<td>Gau</td>
<td>0.96</td>
<td></td>
<td>$g_{KNN}/\sqrt{4\pi}$</td>
</tr>
<tr>
<td></td>
<td>$G_T(K^*)/4\pi$</td>
<td>0.164</td>
<td>$G_T(K^*)/4\pi$</td>
<td>0.506</td>
</tr>
<tr>
<td>$K^*(892)$</td>
<td>Gau</td>
<td>2.50</td>
<td></td>
<td>$G_V(K^*)/4\pi$</td>
</tr>
<tr>
<td></td>
<td>Gau</td>
<td>1.06</td>
<td></td>
<td>$G_T(K^*)/4\pi$</td>
</tr>
<tr>
<td>$K_1(1270)$</td>
<td>GD1</td>
<td>2.12</td>
<td></td>
<td>$G_N(1650)/\sqrt{4\pi}$</td>
</tr>
<tr>
<td>$N(1650)S_{11}$</td>
<td>Dip</td>
<td>2.50</td>
<td></td>
<td>$G_{K(1700)}/\sqrt{4\pi}$</td>
</tr>
<tr>
<td>$N(1700)D_{13}$</td>
<td>Dip</td>
<td>2.12</td>
<td></td>
<td>$G_{K(1700)}/\sqrt{4\pi}$</td>
</tr>
<tr>
<td>$N(1710)P_{11}$</td>
<td>Gau</td>
<td>2.46</td>
<td></td>
<td>$G_N(1710)/\sqrt{4\pi}$</td>
</tr>
<tr>
<td>$N(1720)P_{13}$</td>
<td>Gau</td>
<td>1.59</td>
<td></td>
<td>$G_N(1720)/\sqrt{4\pi}$</td>
</tr>
<tr>
<td>$N(1840)P_{11}$</td>
<td>Gau</td>
<td>1.43</td>
<td></td>
<td>$G_N(1840)/\sqrt{4\pi}$</td>
</tr>
<tr>
<td>$N(1900)P_{13}$</td>
<td>Dip</td>
<td>1.47</td>
<td></td>
<td>$G_{K(1900)}/\sqrt{4\pi}$</td>
</tr>
<tr>
<td>$N(2080)D_{13}$</td>
<td>Dip</td>
<td>1.37</td>
<td></td>
<td>$G_{K(2080)}/\sqrt{4\pi}$</td>
</tr>
<tr>
<td>$N(2090)S_{11}$</td>
<td>Dip</td>
<td>0.89</td>
<td></td>
<td>$G_{K(2090)}/\sqrt{4\pi}$</td>
</tr>
<tr>
<td>$A(1600)P_{01}$</td>
<td>Dip</td>
<td>2.50</td>
<td></td>
<td>$G_{K(2100)}/\sqrt{4\pi}$</td>
</tr>
<tr>
<td>$A(1810)P_{01}$</td>
<td>Gau</td>
<td>1.94</td>
<td></td>
<td>$G_{K(1810)}/\sqrt{4\pi}$</td>
</tr>
</tbody>
</table>
large. This can be seen from the $K^*$ form factor cutoff shown in Table 3, which is relatively soft and, as a consequence, strongly suppresses the $K^*$ contribution.

From Table 3, it appears that different diagrams require different types of HFFs. Physically, this is because as functions of energy and kaon angle contribution of each diagram is unique. Furthermore, at higher energies certain diagrams yield a large contribution, whereas others do not. As a consequence, in order to reproduce the experimental data the form factors required to adjust these contributions should behave differently. This is the reason behind the need for different types of HFFs as listed in Table 3.

The different values of hadronic cutoff for different intermediate states, as shown in Table 3, correspond directly to different required suppression. From our experience,\(^{41,42}\) since the Born terms increase significantly as the energy increases, they need a strong suppression, i.e., a small cutoff. This is obvious from Table 3. The left panel of Fig. 2 elucidates this situation. For the other background terms, i.e., the $t$-channel resonances $K^*(892)$ and $K_1(1270)$, as well as the $u$-channel resonances $\Lambda(1600)P_{01}$ and $\Lambda(1810)P_{01}$, the situation is rather complicated.\(^a\) In principle, their contributions should also be significantly limited in order to produce a reasonable cross-section at higher energies. However, the required suppression is also determined by the angular distributions of cross-section and other polarization observables, which is obvious from the right panel of Fig. 2. Therefore, the combination of a relatively large CC and a strong cutoff (small $\Lambda$), or vice versa, seems to be the most appropriate one, as shown by the CCs and hadronic cutoffs of both $K^*(892)$ and $K_1(1270)$ in Table 3. This is also valid for the $u$-channel resonances $\Lambda(1600)P_{01}$ and $\Lambda(1810)P_{01}$, although their role is different from the role of the $t$-channel resonances. The $t$-channel controls the forward behavior of the observables, whereas the $u$-channel controls the backward one.

\(^a\)Note that in our definition all resonances “resonate” only in the $s$-channel. Therefore, the $t$-channel and $s$-channel resonances belong to the background terms.
In contrast to the Born terms, the contribution of resonance terms typically does not dramatically increase at higher energies. We note that in the covariant formalism used in the present analysis the resonance terms also generate additional backgrounds. Nevertheless, they differ from the Born terms, i.e., the corresponding background amplitude does not violently increase as in the case of Born terms. Thus, the resonance terms usually do not require a strong suppression. The \( N(2090)S_{11} \) would presumably be a special case. The small CC and soft form factor in this case can be interpreted as an indication that the \( N(2090)S_{11} \) plays a less important role in the \( K^+\Lambda \) photoproduction.

Note that the use of HFFs is not the only mechanism to suppress the divergence of the amplitude. In Ref. 43, it is shown that one can also utilize the coupled channel effect for this purpose. Nevertheless, it should be noted that the partial waves calculation in Ref. 43 makes use of 20 nucleon resonances in order to achieve a good agreement with the experimental data. The number of used resonances is therefore twice larger than that used in the present study. Obviously, the number of free parameters in Ref. 43 is also larger. Furthermore, the agreement of the model with experimental data in the case of kaon photoproduction is only excellent at...
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low energies. For $W \geq 1700\text{MeV}$, the discrepancy between model calculation and experimental data starts to become significant, especially in the forward kinematics (see Fig. 37 of Ref. 43).

To check and visualize the result of our calculation, in Fig. 3 we compare the differential cross-sections obtained from the best model (Table 3) with those obtained by using the global dipole, Gaussian and generalized dipole form factors, along with the corresponding experimental data. From Fig. 3, it is clear that the best model can nicely reproduce the experimental data and indeed much better than other models. In the case that experimental data indicate an internal inconsistency, the best model produces a more democratic result (average values) or approaches the data with smaller error bars. Note that experimental data near the threshold energy are more scattered and, as a consequence, indicate more inconsistency than those at higher energies. In this kinematics the latest CEBAF Large Acceptance Spectrometer (CLAS) data\textsuperscript{44} tend to be higher than the previous one,\textsuperscript{45} whereas the former have larger error bars near the threshold. Such situation would certainly increase the complexity of phenomenological studies at threshold, as has been pointed out in Refs. 46 and 47.

The same trend is also exhibited in the case of recoil polarization, target asymmetry and photon asymmetry, as shown in Fig. 4. Here the best model can nicely reproduce the recoil polarization data, except in the high-energy region, where

![Graph](image-url)

Fig. 4. (Color online) Recoil polarization, target asymmetry, and photon asymmetry observables obtained from calculations with different HFF configurations compared with experimental data. Notation of the lines is the same as in Fig. 3. Experimental data are from the CLAS (solid blue squares\textsuperscript{45} and open circles\textsuperscript{44}), GReNoBLE Anneau Accelerateur Laser (GRAAL) (solid red circles\textsuperscript{49} and solid black triangles\textsuperscript{50}) and LEPS (open squares\textsuperscript{51}) collaborations.
other calculations exhibit the same phenomenon. This is understandable, because the model does not include nucleon resonances with $m \approx 2.2$ GeV. In the case of target and photon asymmetries, we can see that the best model can describe experimental data much better than other calculations. Future experiments at Super Photon Ring-8 Gev (SPring-8) or Continuous Electron Beam Accelerator Facility (CEBAF) would be able to put more constraints on the model by measuring the asymmetries at higher energies, where the discrepancy between models becomes more evident.

Finally, in the case of double polarization observables $O_{x'}$, $O_{z'}$, $C_x$ and $C_z$, shown in Fig. 5, the best model displays its superiority. Especially in the case of $C_x$ and $C_z$, where experimental data at high energies are available, the best model can reproduce the data within their error bars.

5.2. Internal consistency of the data

Since the experimental data used in this analysis come from different experiments, it is important to check their internal consistencies. Previous phenomenological studies found that a number of data sets exhibit inconsistencies internally, which therefore increases the difficulty of the model to reproduce experimental data. To this end we can calculate the relative deviation of the data to the theoretical calculation, i.e.,

$$\chi = \frac{O_{\text{exp}} - O_{\text{theor}}}{\Delta O_{\text{exp}}},$$

Fig. 5. (Color online) Energy distribution of the double polarization observables $O_{x'}$, $O_{z'}$, $C_x$ and $C_z$ obtained from calculations with different HFF configurations compared with experimental data. The value of $\cos \theta$ is given in each panel. Notation of the lines is the same as in Fig. 3. Experimental data are from the GRAAL (open circles) and CLAS (open squares) collaborations.
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where $O_{\text{exp}}$ ($O_{\text{theor}}$) is the experimental (theoretical) value of a specific observable and $\Delta O_{\text{exp}}$ is the corresponding experimental uncertainty. Note that the square of Eq. (14) has been used in the present analysis to calculate the $\chi^2/N$ values given in Tables 1–3.

The result is shown in Fig. 6. Although not all data are included for the sake of simplicity, all types of observables are represented in this figure. The average deviations are shown by the solid blue lines, where their values vary from $-0.01$ to $0.80$. The largest deviations are found for the $C_x$ ($0.80$) and $T$ ($-0.62$) observables, whereas the smallest deviation is found for the recoil polarization $P$ ($-0.01$). These deviations are much smaller than those found in previous studies.\(^{41,53}\) We have also cross-checked this finding by refitting the data with $|\chi| \leq 1, 2, 3$ and $4$, step by step. We found that the value of $\chi^2/N$ obtained in all steps do not differ significantly from that obtained by fitting all available data. The calculated observables also show the same result. This happens, because the number of data with $|\chi| \leq 1$ is sufficiently large. On the other hand, the result of this cross-check corroborates the conclusion drawn from Fig. 6, i.e., within the best model used in this analysis all experimental data seem to be consistent. Note that in this analysis we do not use the Spectrometer Arrangement for Photon Induced Reactions (SAPHIR) data,\(^{55,56}\) since it has been shown\(^ {54}\) that these data show an inconsistency with the CLAS data.\(^ {44,45}\)

5.3. Differential cross-section at forward angles

It has been shown in Ref. 41 that at forward angles theoretical models and experimental data show a large variance (see Fig. 1 of Ref. 41), in spite of the fact that
Theoretical prediction of the hypernucleus production cross-section depends sensitively on the elementary amplitude at this kinematics. Therefore, a better determination of the elementary amplitude at this kinematics becomes an important agenda of theoretical and experimental investigations at present and in the future.

We observe that the currently available experimental data do not reach the forward kinematics. Only a number of older data points can reach $\theta_K \approx 6^\circ$. In the case of the CLAS and SAPHIR detectors this is apparently understandable as an intrinsic problem. However, this problem would not appear for the experiments at Mainz Microtron (MAMI) and SPring-8, where forward measurements of other processes have been successfully performed.

In the previous study, we have investigated the effect of different global HFFs on the calculated differential cross-section at forward angles. It is found that the predicted cross-section at this kinematics depends sensitively on the choice of the

![Graph](image-url)

Fig. 7. (Color online) Variation of the differential cross-section at forward angles due to the variation of the HFF cutoffs (left panels) and the hadronic CCs (right panels). Solid lines indicate the result from the best form factor configuration, whereas in the left panels the dash–dotted black lines (the dashed green lines) are obtained by increasing (decreasing) the value of the $K^*$ HFF cutoff by 10%. In the right panels the dash–dotted black lines (the dashed green lines) are obtained by increasing (decreasing) the value of the $K^*$ CCs $G_K^V$, and $G_K^T$, by 20%. Solid blue squares and open circles show the experimental data taken from the CLAS collaboration. Open squares indicate the older data that are not used in the fitting process. The corresponding value of the total c.m. energy in GeV is written in each panel.
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form factor (see Fig. 8 of Ref. 1). Since the best model obtained in the present analysis has a specific HFF combination as shown in Table 3, we may vary their cutoffs as well as the related CCs in order to investigate the sensitivity of the differential cross-section at forward kaon angles. We found that in most cases the cross-section is insensitive to these variations. Only in the case of $K^*$ intermediate state we found sizable effect, though relatively small at $\theta_K \approx 0$. This is shown in Fig. 7, where we compare the differential cross-section obtained by different $K^*$ cutoffs ($\pm 10\%$ from its original value) and CCs ($\pm 20\%$ from their original values) with experimental data. Only at larger angles ($\gtrsim 30^\circ$) the effect becomes significantly large. Therefore, we may conclude that the choice of HFF determines the cross-section behavior at threshold, whereas variation of the hadronic cutoffs and CCs influence this behavior slightly.

6. Summary and Conclusion

We have investigated the kaon photoproduction process $\gamma p \rightarrow K^+\Lambda$ by searching for the best form factor combination that yields the smallest $\chi^2/N$. Instead of performing more than $6 \times 10^9$ fit combinations, we have proposed a simple fit strategy to find the best model with the smallest $\chi^2/N$. This was performed by grouping the terms according to their properties during the fit process. The best model was obtained by using the dipole form factor for the Born, $N(1650)S_{11}$, $N(1700)D_{13}$, $N(1900)P_{13}$, $N(2080)D_{13}$, $N(2090)S_{11}$ and $N(2100)P_{11}$ terms, whereas for the $K^*(892)$, $N(1710)P_{11}$, $N(1720)P_{13}$, $N(1840)P_{11}$ and the two hyperon resonances $\Lambda(1600)P_{11}$ and $\Lambda(1810)P_{01}$, the model utilizes the Gaussian form. Only for the $K_1(1270)$ exchange, the model requires the generalized dipole form factor with $r = 1$. With $\chi^2/N = 2.13$, the model can nicely reproduce the available experimental data with different types of observables. From the model point of view, the experimental data used are found to be internally consistent, since the average of their deviations is relatively small. We have also found that variation of the form factor cutoff and hadronic CCs only slightly changes the magnitude of the differential cross-section at forward angles.

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